### **Interactive GRE Math Flashcards**

These GRE math flashcards feature screenshots from my video course.

www.GreenlightTestPrep.com



#### **Interactive GRE Math Flashcards**

If, at any time, you'd like to watch the video related to a certain flashcard, just click on the link at the top of that page, and you'll be taken to the corresponding video\*

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\*Many of the course videos are free, but you must register an account to have access to all videos



#### **Interactive GRE Math Flashcards**

# TABLE OF CONTENTS

topic - slide

quantitative comparison strategies - 4

arithmetic - 11

powers and roots - 34

algebra and equation-solving - 50

word problems - 87

geometry - 107

integer properties - 130

statistics - 145

counting - 152

probability - 161

data interpretation - 171



(watch the entire video <a href="here">here</a>)

- The first 7-9 questions of each math section will be QC questions
- Allot approximately 1¼ to 1½ minutes per question
- Given Quantity A and Quantity B
- Compare the two quantities and choose:
  - A if Quantity A is greater
  - B if Quantity B is greater
  - c if the two quantities are equal
  - the relationship cannot be determined from the information given



# **QC Strategy - Approximation Techniques**

Do not perform more calculations than are necessary

Compare quantities in parts

$$A \times B = 2A \times \frac{1}{2}B$$

area = 
$$\pi(25)$$
  
=  $(3^+)(25)$   
=  $75^{++}$  "bigger than"



(watch the entire video <a href="here">here</a>)

## **QC Strategy - Matching Operations**

#### Acceptable operations

- Add <u>any</u> value to both quantities
- Subtract any value from both quantities
- Multiply both quantities by a positive value
- Divide both quantities by a positive value

Do not multiply or divide both quantities by a variable unless you are certain that the variable has a <u>positive</u> value



# **QC Strategy - Plugging in Numbers**

Variable(s) in the quantities consider plugging in values

#### Drawback

Without two contradictory results, you cannot be certain of the correct answer

#### Benefit

You can quickly narrow the answer choices down to two options

 great for questions where you don't know how to proceed!

"Nice" numbers: 0, 1, -1, 
$$\frac{1}{2}$$
,  $-\frac{1}{2}$ , 10, -10



# **QC Strategy – Looking for Equality**

Plugging in numbers?



Is there a value for the variable that makes the two quantities equal?



(watch the entire video <a href="here">here</a>)

# **QC Strategy – Number Sense**

 Some Quantitative Comparison questions can be solved quickly by applying some number sense



(watch the entire video <a href="here">here</a>)

# **QC Strategy - Miscellaneous Tips**

- Avoid making unnecessary computations
- Do not select D if the comparison does not contain unknown values
- Geometry figures are not necessarily drawn to scale (unless stated otherwise)
- Pay close attention to the shared information



# **Real number**: any number that can be shown on the number line

$$a+b=b+a$$

$$a \times b = b \times a$$

$$(a+b)+c=a+(b+c)$$

$$(a \times b) \times c = a \times (b \times c)$$

$$a(b+c)=ab+ac$$

$$1 \times a = a$$

$$a \div 1 = a$$

$$\mathbf{a} \times \mathbf{0} = \mathbf{0}$$

$$a + 0 = a$$

$$a \div a = 1$$
 (as long  $a \ne 0$ )



## Adding a positive number

Move right along the number line

## Subtracting a positive number

Move left along the number line

## Adding a negative number

• Same as subtracting a positive number a + (-b) = a - b

## <u>Subtracting a negative number</u>

• Same as adding a positive number a - (-b) = a + b



```
(positive) x or ÷ (positive) = positive
(negative) x or ÷ (positive) = negative
(positive) x or ÷ (negative) = negative
(negative) x or ÷ (negative) = positive
```

2 like signs produce a positive number2 different signs produce a negative number



**Parentheses** 

Exponents

Multiplication

Division

Addition

Subtraction

Brackets

Exponents

Division

Multiplication

Addition

Subtraction

**absolute value**: a number's distance from zero on the number line





Next digit is 0, 1, 2, 3 or  $4 \Rightarrow$  round down Next digit is 5, 6, 7, 8 or  $9 \Rightarrow$  round up

#### Adding and subtracting decimals

- Line up the decimals
- Add additional zeros (or assume there are zeros)

#### Multiplying decimals

- Find the total number of digits to the right of each decimal
- Ignore the decimals and find the product
- Take product and move the decimal place to the left

#### **Dividing decimals**

- Move both decimals until the divisor becomes an integer
- Divide, keeping the decimal in the same location



# Multiplying by powers of 10

• Move the decimal 1 space right for each zero

# Dividing by powers of 10

• Move the decimal 1 space left for each zero



# Equivalent fractions

$$\frac{1}{2}=\frac{5}{10}$$

$$\frac{1}{2} = \frac{5}{10}$$
  $\frac{7}{9} = \frac{14}{18}$   $\frac{3}{5} = \frac{30}{50}$ 

$$\frac{3}{5} = \frac{30}{50}$$

 Create equivalent fractions by multiplying/dividing the numerator and denominator by the same number

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{24}{36} = \frac{24 \div 4}{36 \div 4} = \frac{6}{9}$$

$$\frac{10}{11} = \frac{10 \times 7}{11 \times 7} = \frac{70}{77}$$

$$\frac{35}{45} = \frac{35 \div 5}{45 \div 5} = \frac{7}{9}$$

# Converting entire fractions into mixed numbers



- Determine how many times the denominator divides into the numerator (this becomes the whole number portion)
- The remainder becomes the numerator of the new fraction.
- The denominator remains the same



# Converting mixed numbers into entire fractions

- Multiply the whole number by the denominator, and add the product to the numerator
- The result becomes the new numerator and the denominator remains the same

$$6\frac{1}{3} = \frac{19}{3}$$

$$11\frac{2}{7} = \frac{79}{7}$$



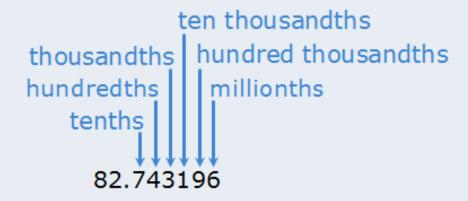
# Converting fractions to decimals

$\frac{1}{2}$	0.5	
1 3	~ 0.333	
$\frac{1}{4}$	0.25	
$\frac{1}{5}$	0.2	
$\frac{1}{6}$	~ 0.166	
$\frac{1}{7}$	~ 0.14	
18	0.125	
$\frac{1}{9}$	~ 0.11	



# Converting decimals to fractions

- Find the place value of the last digit
- Write a fraction with that place value as the denominator





$$n=\frac{n}{1}$$

 $\frac{n}{0}$  is undefined

$$\frac{n}{n} = 1$$
 (as long as  $n \neq 0$ )

$$\frac{1}{\frac{a}{b}} = \frac{b}{a} \quad \text{(as long as } a \neq 0 \text{ and } b \neq 0\text{)}$$

$$\frac{a}{b} \times \frac{b}{a} = 1$$
 (as long as  $a \neq 0$  and  $b \neq 0$ )

The reciprocal of  $\frac{a}{b}$  is  $\frac{b}{a}$ 



- Bigger numerator → bigger value
- Smaller numerator → smaller value
- Bigger denominator → smaller value
- Smaller denominator → bigger value

Increase numerator and denominator by same amount → fraction approaches 1



## Adding and subtracting fractions

- Create equivalent fractions with the same denominator
- Add/subtract the numerators
- Keep the denominator the same

## Multiplying fractions

- · Multiply numerators, and multiply denominators
- · Convert to entire fractions before multiplying
- When possible, simplify fractions before multiplying
- When possible, "cross simplify" before multiplying

# **Dividing fractions**

Multiply by the reciprocal of the divisor



$$\frac{abc}{def} = \frac{a}{d} \times \frac{b}{e} \times \frac{c}{f}$$

$$\frac{a+b+c}{d+e+f} \neq \frac{a}{d} + \frac{b}{e} + \frac{c}{f}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \left| \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c} \right|$$

$$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$

$$\frac{a+b}{c+d} = \frac{a}{c+d} + \frac{b}{c+d}$$

$$\frac{a}{b} \times b = a$$
 (as long as  $b \neq 0$ )

If 
$$\frac{a}{b} = \frac{c}{d}$$
 then  $ad = bc$ 



_		
$\frac{1}{2}$	0.5	50%
1 3	~ 0.333	~ 33.3%
$\frac{1}{4}$	0.25	25%
1 5	0.2	20%
$\frac{1}{6}$	~ 0.166	~ 16.6%
$\frac{1}{7}$	~ 0.14	~ 14%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{9}$	~ 0.11	~ 11.1%

# Conversions (decimal to percent)

 Move decimal two places to the right

# Conversions (percent to decimal)

 Move decimal two places to the left



# The part is some percent of the whole

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

If 
$$\frac{a}{b} = \frac{c}{d}$$
 then  $ad = bc$ 

p percent of x is 
$$y \rightarrow \left(\frac{p}{100}\right)(x) = y$$



# 10 percent of y

move decimal 1 space to the left

# 1 percent of y

move decimal 2 spaces to the left

What is 15 percent of 62?





% change = 
$$\frac{\text{change}}{\text{original value}} \rightarrow \text{(then rewrite as a percent)}$$

% change = 
$$\frac{\text{change} \times 100}{\text{original value}}$$

$$new = \left(1 \pm \frac{percent \ change}{100}\right) \times original$$



### Compound interest

final = 
$$P\left(1 + \frac{r}{c}\right)^{nc}$$

P = principal

r =annual interest rate (as a decimal)

c = number of "compoundings" per year

n = number of years

simple interest
interest =(principal)(rate)(time)

 For short time periods, consider incremental calculations e.g.,



"For every x there are  $y \dots "$   $\Rightarrow$  ratio question

## **Equivalent ratios**

If 
$$\frac{a}{b} = \frac{c}{d}$$
 then  $ad = bc$ 

# Portioning into ratios

- Add the terms in the ratio and let the sum = T
- Divide the total quantity into T equal parts
- Divide the T equal parts into the target ratio



# **Combining Ratios**

### Strategy 1

- Find equivalent ratios until there are matching terms
- Combine

## Strategy 2

- Solve one ratio
- Apply results to the other ratio



• Ratios can be used to solve simple rate questions



base 
$$\rightarrow 2^5$$
 exponent

1 raised to any power is equal to 1

0 raised to any nonzero power is equal to 0

Any nonzero number raised to the power of 0 is equal to 1

Any number, x, raised to the power of 1 is equal to x

An odd exponent preserves the sign of the base

An even exponent always yields a positive result

\* as long as the base  $\neq 0$ 



# **Exponential Growth**

#### Positive bases

If x > 1, then the value of  $x^n$  increases as n increases

If 0 < x < 1, then the value of  $x^n$  approaches zero as n increases

### Negative bases

If x < -1, then the magnitude of  $x^n$  increases as n increases, but the sign oscillates

If -1 < x < 0, then the magnitude of  $x^n$  decreases as n increases



# Squaring Integers Ending in 5

$$7 \times 8 = 56$$
 $10 \times 11 = 110$ 
 $75^2 = 5625$ 
 $105^2 = 11025$ 

### <u>Technique</u>

- Let n be the number before the 5
- Write the product of n and n+1, followed by 25



# Quotient law

$$\frac{\mathbf{X}^a}{\mathbf{X}^b} = \mathbf{X}^{a-b}$$

#### Product law

$$(x^a)(x^b) = x^{a+b}$$

Power of a power law

$$\left(X^{a}\right)^{b}=X^{ab}$$



$$X^{-n} = \frac{1}{X^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$



Power of a product law

$$\left(X^{a}Y^{b}\right)^{n}=X^{an}Y^{bn}$$

Power of a quotient law

$$\left(\frac{X^a}{Y^b}\right)^n = \frac{X^{an}}{Y^{bn}}$$

Combining bases law

$$x^n y^n = (xy)^n$$

Combining bases law

$$\frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$$



What is the units digit of 53<sup>35</sup>?

$$53^{1} = 53 
53^{2} = ---9 
53^{3} = ---7$$

$$53^{4} = ---1$$

$$53^{5} = ---3$$

$$53^{6} = ---9$$

$$53^{7} = ---7$$

$$53^{8} = ---1$$

$$53^{35} = ---7$$

$$53^{35} = ---7$$

$$53^{35} = ---7$$

$$53^{35} = ---7$$

$$53^{35} = ---7$$

$$53^{35} = ---7$$



 $\sqrt{n}$  = a number (greater than or equal to zero) that, when squared, equals n

#### **Properties**

- If n < 0,  $\sqrt{n}$  has no real value
- If  $n \ge 0$ , then  $\sqrt{n} \ge 0$

$$\sqrt{x^2} = |x|$$

If 
$$0 < x < 1$$
 then  $\sqrt{x} > x$   
If  $x > 1$  then  $\sqrt{x} < x$ 



 $\sqrt[n]{n}$  = a number that, when raised to the power of r, equals n

A root will have, at most, 1 value

$$\sqrt[n]{X} = X^{\frac{1}{n}}$$



$$\left(\sqrt[n]{x}\right)\left(\sqrt[n]{y}\right) = \sqrt[n]{xy}$$

$$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$$



#### Simplifying Square Roots

4, 9, 16, 25, 36, 49, 64, 81, 100, 121, . . .

$$\sqrt{700} = \sqrt{100 \times 7}$$

 $\sqrt{700} = \sqrt{100 \times 7}$  • Rewrite the number inside the root as the product of a perfect square and some other number

$$= \sqrt{100} \times \sqrt{7}$$

=  $\sqrt{100} \times \sqrt{7}$  • Rewrite the root as the product of 2 roots

$$=10\sqrt{7}$$

Simplify the root of the perfect square



# **Operations with Roots**

Multiply the parts outside the root and multiply the parts inside the root

Divide the parts outside the root, and divide the parts inside the root

$$\sqrt{a} + \sqrt{b} = \sqrt{a+b}$$

$$n\sqrt{a+b} = \sqrt{na+nb}$$



$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$
  $\Rightarrow 32^{\frac{3}{5}} = \sqrt[5]{32^3}$ 

$$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$$
  $\Rightarrow 32^{\frac{3}{5}} = (\sqrt[5]{32})^3 = (2)^3 = 8$ 

$$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$$
  $81^{\frac{3}{4}} = (\sqrt[4]{81})^3 = 3^3 = 27$ 



### **Equations with Exponents**

- 1) Rewrite with equal bases
- 2) Apply following rule
- 3) Solve resulting equation

If 
$$b^x = b^y$$
 then  $x = y$   
 $(b \neq 0, 1, -1)$ 



## "Fixing" the Denominator

$$\frac{6\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

Multiply top and bottom by the root in the denominator

$$\frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} - 2\sqrt{5}} \times \frac{\sqrt{3} + 2\sqrt{5}}{\sqrt{3} + 2\sqrt{5}}$$

Multiply top and bottom by the conjugate of the denominator



#### Definitions

expression: collection of one or more terms combined using addition and/or subtraction

examples: 
$$w^3 - 3x^2 + 5y$$
  
 $x - 1$   

$$\frac{2x^4}{5} + \frac{1}{y^3} - 5x^2y + x - 3y + 9$$

monomial: expression with 1 term

examples: 14, 5x,  $8xy^3$ ,  $\frac{jk}{5m^3}$ 

binomial: expression with 2 terms

examples:  $x^2 + 3y$ W - 8

polynomial: expression with 1 or more terms



### Simplifying Expressions

Like terms can be combined (added/subtracted)

e.g., 
$$2x + 7x = 9x$$

To add expressions in parentheses, remove the parentheses

e.g., 
$$(3x-2y)+(x-7y)=3x-2y+x-7y$$

To subtract expressions in parentheses, add the "opposites"

e.g., 
$$(3x-2y)-(x-7y)=(3x-2y)+(-x+7y)$$



Multiply members of the same "family"

e.g., 
$$(5y^3)(4y^4) = 20y^7$$

Multiply each term in the parentheses by the term in front

e.g., 
$$3(2x+5) = 6x+15$$



#### <u>Multiplying two binomials</u>

First 
$$(x+2)(x+7) = x^2 + 7x + 2x + 14$$
  
Outer  $= x^2 + 9x + 14$ 

Inner

Last 
$$(3y-4)(2y-5) = 6y^2 - 15y - 8y + 20$$
  
=  $6y^2 - 23y + 20$ 

$$(2x+y)(x-7y) = 2x^2 - 14xy + xy - 7y^2$$
$$= 2x^2 - 13xy - 7y^2$$



### Multiplying two binomials

First

Outer

Inner

Last

$$(a + b)^2 = (a + b)(a + b)$$
  
=  $a^2 + ab + ab + b^2$   
=  $a^2 + 2ab + b^2$ 

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^{2} = (a-b)(a-b)$$

$$= a^{2} - ab - ab + b^{2}$$

$$= a^{2} - 2ab + b^{2}$$

$$(a-b)^{2} = a^{2} - 2ab + b^{2}$$



#### <u>Greatest Common Factor Factoring</u>

- Find the greatest common factor (divisor) of all terms
- Place the greatest common factor in front of the parentheses
- Determine which terms must be inside the parentheses to get the desired product



#### <u>Difference of Squares Factoring</u>

Watch out for differences of squares

$$a^2 - b^2 = (a + b)(a - b)$$



## **Quadratic Polynomial Factoring**

$$x^{2} + nx + p = (x + a)(x + b)$$

$$a + b$$

$$ab$$



# Factoring – Putting it all Together

- 1. Factor out the greatest common factor
- Factor further (if possible)

Example: 
$$2x^6 - 2x^2 = 2x^2(x^4 - 1)$$
  
=  $2x^2(x^2 + 1)(x^2 - 1)$   
=  $2x^2(x^2 + 1)(x + 1)(x - 1)$ 



## Simplifying Rational Expressions

$$\frac{x^3 + 4x^2 + 3x}{x^3 + 2x^2 - 3x} = \frac{x(x^2 + 4x + 3)}{x(x^2 + 2x - 3)}$$
$$= \frac{x(x+3)(x+1)}{x(x+3)(x-1)}$$
$$= \frac{x+1}{x-1}$$

$$\Rightarrow \frac{x^3 + 4x^2 + 3x}{x^3 + 2x^2 - 3x} = \frac{x + 1}{x - 1}$$
 for all values of x for which both expressions are defined



## Golden Rule of Equation Solving

What you do to one side of the equation, you must do to the other side

- Isolate the variable by performing the same operations to both sides
- "solution" = "root"



$$\frac{x}{10} + \frac{4}{5} = \frac{x}{12} + 1$$

$$\frac{60}{1}\left(\frac{x}{10} + \frac{4}{5}\right) = \frac{60}{1}\left(\frac{x}{12} + 1\right)$$

$$\frac{60x}{10} + \frac{240}{5} = \frac{60x}{12} + \frac{60}{1}$$

$$6x + 48 = 5x + 60$$

$$x + 48 = 60$$

$$x = 12$$

Multiply both sides by the least common multiple of the denominators



$$\frac{7}{6x-6} = \frac{3}{2x+2}$$

$$\frac{7}{6x-6} = \frac{3}{2x+2} \qquad \text{If } \frac{a}{b} = \frac{c}{d} \text{ then } ad = bc$$

$$7(2x+2)=3(6x-6)$$

$$14x + 14 = 18x - 18$$

$$14 = 4x - 18$$

$$32 = 4x$$

$$8 = x$$



$$ax^2 + bx + c = 0 (a \neq 0)$$

- Most (all) quadratic equations can be solved by factoring
- Solvable quadratic equations will have 1 or 2 unique solutions (roots)

Quadratic formula  

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(b^2 - 4ac) < 0 \implies$$
 no solution exists

$$(b^2 - 4ac) \ge 0$$
  $\Rightarrow$  solution exists



## 2 Equations with 2 Unknowns

#### Substitution Method

- Solve one equation for one variable
- Take the other equation and replace the chosen variable with its equivalent expression from the first equation
- Solve for the variable
- Plug the solution into any equation to solve for the other variable



### 2 Equations with 2 Unknowns

#### Elimination Method

- Manipulate equations until you have matching coefficients for one variable
- Add or subtract the 2 equations to eliminate one variable
- Solve for the remaining variable
- Plug the solution into any equation to solve for the other variable



#### Number of Solutions

- Solve as usual
  - find solution 📦 1 solution
  - identical equations infinite solutions
  - $-0x + 0y = nonzero value \implies zero solutions$



#### Solving 3 Equations with 3 Unknowns

- 1) Solve one equation for one variable
- Take the other two equations and replace the chosen variable with its equivalent expression from the first equation
- 3) Solve for the two remaining variables
- 4) Plug the solution into any equation to solve for the third variable

or

1) Solve using the elimination method



#### **Equations with Square Roots**

#### Square root

- 1) Eliminate square root by squaring both sides
- 2) Solve for variable
- 3) Check for extraneous roots

#### n<sup>th</sup> root

- 1) Raise both sides by power of n
- 2) Solve for variable
- 3) If *n* is even, check for extraneous roots



## **Equations with Exponents**

- 1) Rewrite with equal bases
- 2) Apply following rule
- 3) Solve resulting equation

If 
$$b^x = b^y$$
 then  $x = y$   
 $(b \neq 0, 1, -1)$ 



## **Equations with Absolute Value**

1) Apply rule 
$$|X| = a \Rightarrow \begin{cases} X = a \\ X = -a \end{cases}$$

- 2) Solve resulting equations
- 3) Check for extraneous roots



### **Strange Operators**

• Use the "recipe" to evaluate



# Solving Inequalities

- Adding and subtracting to/from both sides does not affect the inequality
- Multiplying and dividing both sides by a positive number does not affect the inequality
- Multiplying and dividing both sides by a negative number reverses the inequality



# combining inequalities

$$W < X$$

$$X < Y$$

$$W < X < Y \implies W < Y$$

Rewrite inequalities facing the same direction before trying to combine



## adding inequalities

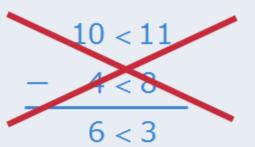
$$A < B$$
 Al < \$15  
+  $C < D$  + Bob < \$10  
 $A + C < B + D$  Al + Bob < \$25

The inequality signs must face the same direction before adding



## subtracting inequalities

$$10 < 17$$
 $- 9 < 10$ 
 $1 < 7$ 



Do not subtract inequalities

Do not multiply inequalities

Do not divide inequalities



$$|x| < a \implies -a < x < a$$
 (where a is positive)

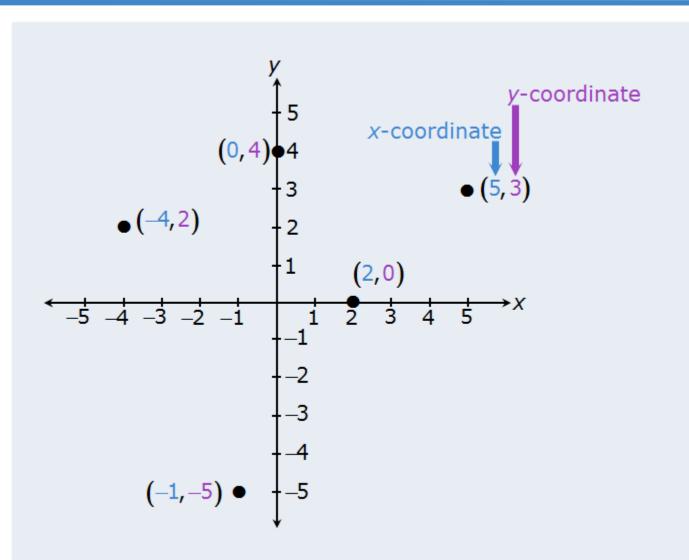
$$|x| > a \implies x > a$$
 or  $x < -a$  (where a is positive)



## Quadratic Inequalities

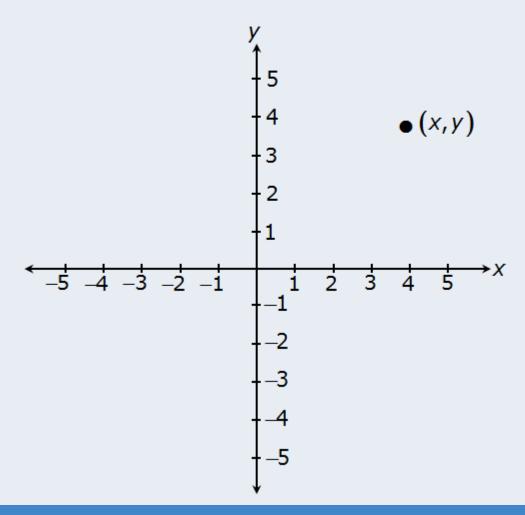
- Set the expression to equal zero
- Find solutions and record on number line
- Test number from each region
- Solve the inequality







• Every point in the coordinate plane is defined by a unique ordered pair of numbers (x, y)



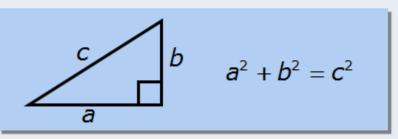


### **Distance Between Two Points**

Apply formula

Distance between points 
$$(x_1, y_1)$$
 and  $(x_2, y_2)$  is  $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ 

Sketch and use Pythagorean Theorem



## **Graphing Lines**

- In the coordinate plane, a line is a set of points such the coordinates of each point satisfy the given equation
- If the coordinates of a point satisfy the equation, then that point will lie on the line
- If the coordinates of a point do not satisfy the equation, then that point will not lie on the line

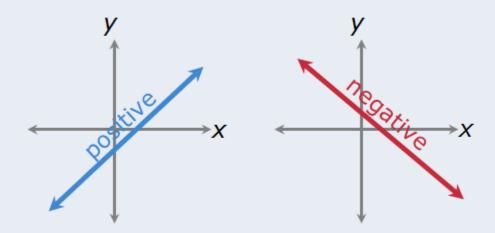


- The graph of the line x=k will be a **vertical** line where all of the points have x-coordinates equal to k
- The graph of the line y=k will be a **horizontal** line where all of the points have y-coordinates equal to k



Given
$$(x_1, y_1)$$
 and  $(x_2, y_2)$   
slope =  $\frac{y_1 - y_2}{x_1 - x_2}$ 

$$slope = \frac{rise}{run}$$



As the magnitude of the slope increases, the line gets steeper



*x*-intercept: *x*-coordinate of point where the line intersects the x-axis

• To find the x-intercept, plug y = 0 into the equation

y-intercept: y-coordinate of point where the line intersects the y-axis

• To find the y-intercept, plug x = 0 into the equation



# Slope y-intercept form

$$y = mx + b$$
  
slope  $y$ -intercept



### Writing equations from two given points

$$y = mx + b$$

- 1) Find the slope (m) of the line
- 2) Plug the value of m into the slope y-intercept equation
- 3) Plug the coordinates of one point into the equation
- 4) Solve for b
- 5) Write the equation in slope y-intercept form

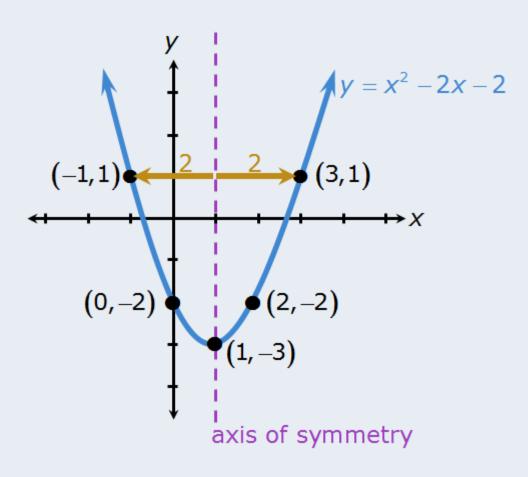
### <u>Practice</u>

$$(1,-2)$$
 and  $(4,7) \Rightarrow y = 3x - 5$ 

$$(4,-9.5)$$
 and  $(-2, 5.5) \Rightarrow y = -\frac{5}{2}x + \frac{1}{2}$ 



$$y = ax^2 + bx + c$$





### Introduction to Word Problems

- 1. Understand the question and any restrictions
- 2. Consider testing the answer choices
- 3. Assign variables
- 4. Create an equation
- 5. Solve the equation (if necessary)
- 6. Reread question and confirm required value



## **Strategy for Testing Answer Choices**

- Test answer choices, beginning with C
- Eliminate impossible answer choices



## **Assigning Variables**

- Consider assigning the variable to the target value
- Consider writing a "word equation"
- It is often best to assign the variable to the smallest value
- Assign descriptive variables
- Look for relationships



## Writing Equations

- Write a "word equation"
- Replace with algebraic expressions
- Solve for variable
- Reread the question and confirm required value

```
profit = revenue – cost
```

total cost = price per item × quantity purchased

total earnings = pay rate × time worked



## Using More than 1 Variable

- Begin with one variable, but change to more variables if there are complex relationships between the unknown values
- n variables requires n equations



## **Past & Future Age Questions**

- Create table with given times
- Create equation(s)
- Ensure that you have obtained the required information



$$distance = rate \times time$$

$$rate = \frac{distance}{time}$$

$$time = \frac{distance}{rate}$$

$$distance = rate \times time$$

The time units must match before multiplying

$$time = \frac{distance}{rate}$$

The distance units must match before dividing



$$average \ speed = \frac{total \ distance \ traveled}{total \ time}$$

Assign variables if necessary



## Multiple Trips and/or Multiple Travelers

- Consider possible word equations
- Use the equation with favorable variables



## Shrinking/Expanding Gaps

- Observe the outcome after 1 unit of time
- Determine shrink/expansion rate
- Apply the rate to the question



## **Work Questions**

$$output = rate \times time$$

$$rate = \frac{output}{time}$$

$$time = \frac{output}{rate}$$



Find the output rates

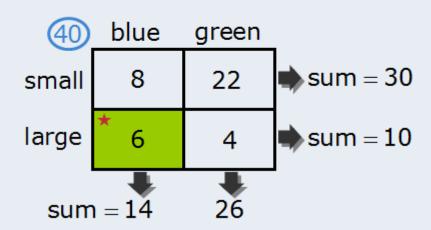
 Add rates when there are two or more contributors (machines, workers, etc.)



## **Double Matrix Method - Example**

In a shipment of 40 toys, each toy is either blue or green, and each toy is either large or small. In total, there are 30 small toys, and there are 14 blue toys. If the shipment contains 22 toys that are both small and green, how many toys are both large and blue?

- A) 4
- **B** 6
  - C) 10
  - D) 12
  - E) 14





## **3-Criteria Venn Diagrams**

- Draw 3 overlapping sets
- Fill in regions from the middle outwards



 Recursive definitions of sequences typically require us to start at the beginning of the sequence



$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

The number of integers from x to y inclusive = y - x + 1



### **Growth Tables**

- For incremental growth or decline draw a table
- Note changes for every time period



## **Mixture Questions**

- Sketch the solution(s) with the parts separated
- Combine like parts



#### Variables in the Answer Choices

#### Algebraic approach

Translate the information into an expression

#### Input-output approach

- Choose value(s) for the given variable(s)
- Use those values to calculate the required output
- Use the same values to evaluate each answer choice, and look for a matching output

Both strategies usually work

The algebraic approach is typically faster



## **Tips for the Algebraic Approach**

- Use real numbers to determine the required operations to reach a certain goal
- Apply those operations to the given variables
- Try writing the expression in different ways



## **Tips for the Input-Output Approach**

- Use small numbers and prime numbers
- Use different numbers
- In most cases, avoid using 0 and 1
- Avoid numbers that appear in the question
- Some numbers allow us to quickly eliminate answer choices



line: a straight path that extends without end in both directions



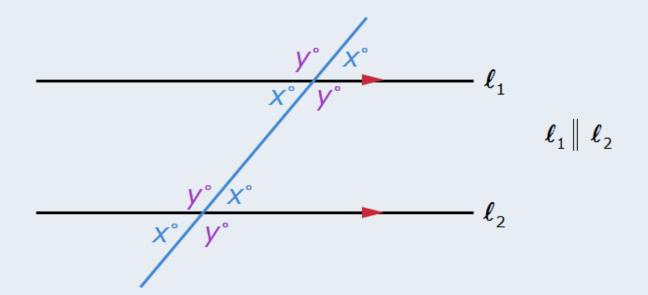
AB: line segment

AB: <u>length</u> of line segment AB (e.g., DE=7)



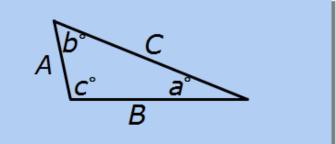
Angles on a line add to 180°

Opposite angles are equal





# Angles in a triangle add to 180°



If a < b < c then A < B < C

The sum of the lengths of any two sides of a triangle must be greater than the third side.

Given lengths of sides A and B

$$|A - B| < 3^{rd}$$
 side  $< A + B$ 



# **Assumptions about Geometric Figures**

- Lines that appear straight can be assumed to be straight
- Angles are greater than 0 degrees
- Do not make assumptions about angle measurements
- Do not make assumptions about parallelism
- Unless otherwise indicated, ∠ABC refers to the smaller angle



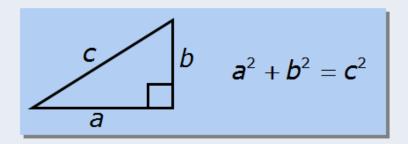
- An isosceles triangle has 2 equal sides and 2 equal angles
- An equilateral triangle has 3 equal sides and 3 equal angles (60° each)

$$Area = \frac{base \times height}{2}$$

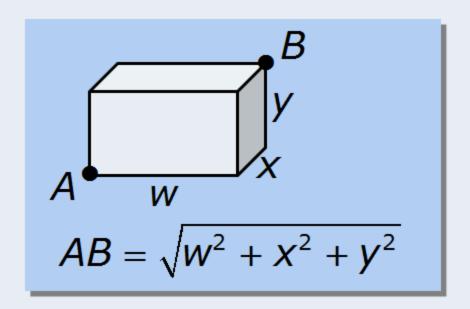
Area = 
$$\frac{\sqrt{3} \times (\text{side})^2}{4}$$

• The altitudes of isosceles triangles and equilateral triangles bisect the base

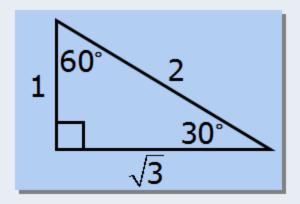


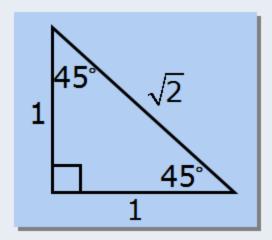


- Watch out for Pythagorean triples (and their multiples)
  - 3-4-5
  - 5-12-13
  - 8-15-17
  - 7-24-25



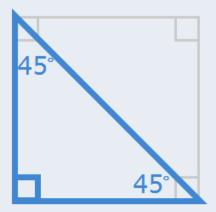




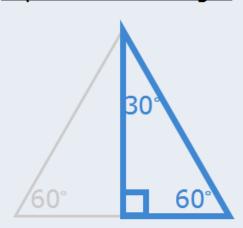




## <u>Square</u>



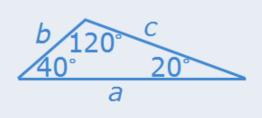
# **Equilateral Triangle**

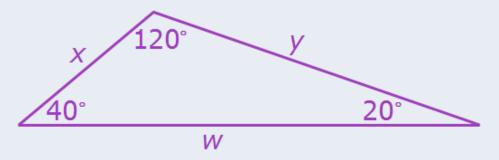


Watch out for special right triangles "hiding" in squares and equilateral triangles



# Similar triangles have the same 3 angles in common





$$\frac{a}{W} = \frac{b}{X} = \frac{c}{y}$$

With similar triangles, the ratio of any pair of corresponding sides is the same



## parallelogram

opposite sides parallel



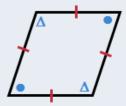
#### rectangle

- opposite sides parallel
- all angles are 90°



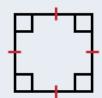
#### rhombus

- opposite sides parallel
- all sides are equal



#### square

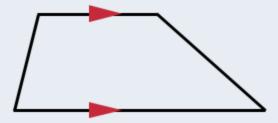
opposite sides parallel





# trapezoid

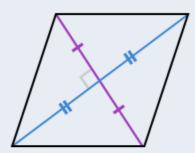
• 2 sides parallel





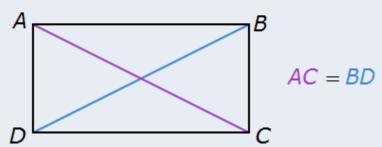
## Rhombus (and square)

• diagonals are perpendicular bisectors

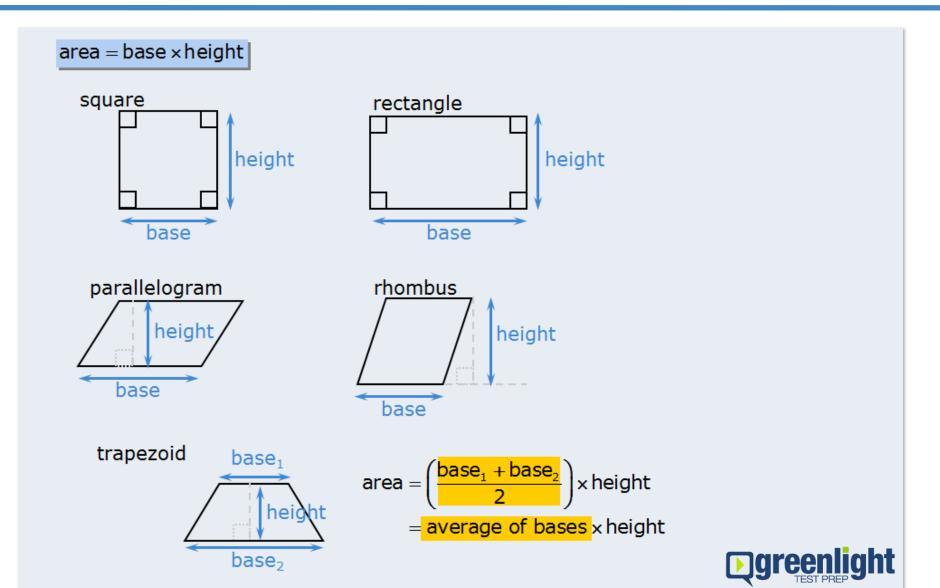


## Rectangle (and square)

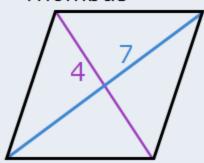
• diagonals are equal length







### rhombus

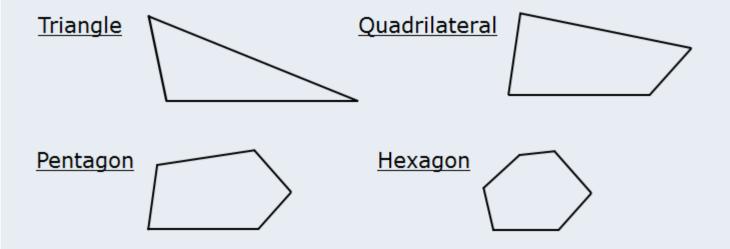


$$area = \frac{diagonal_1 \times diagonal_2}{2}$$

$$area = \frac{4 \times 7}{2}$$
$$= \frac{28}{2}$$
$$= 14$$

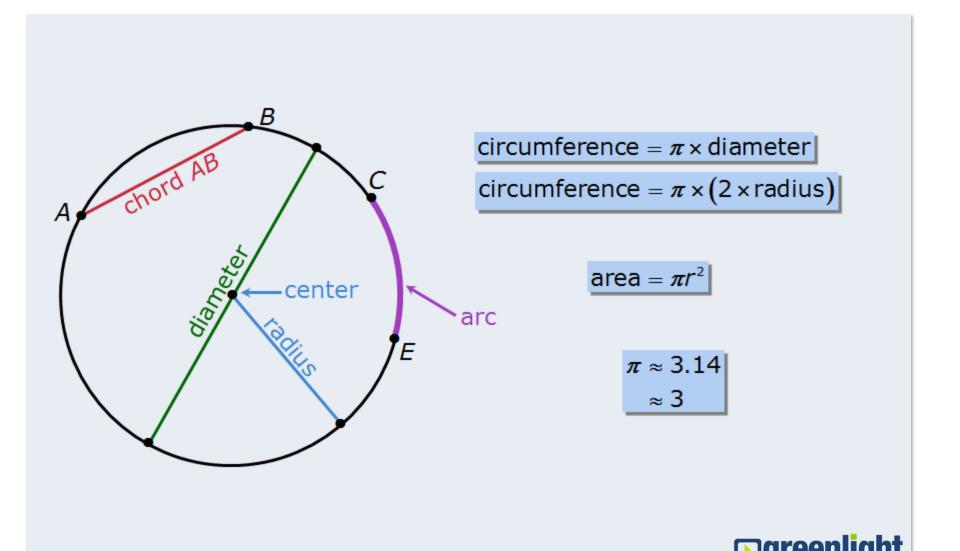


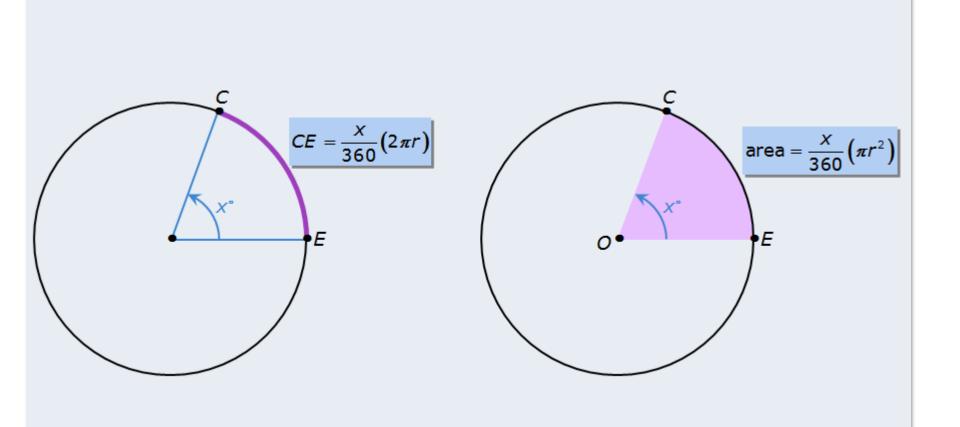
- <u>Polygon</u>: Closed figure formed by 3 or more line segments
- Regular polygon: equal sides and equal angles



The sum of the interior angles in an N-sided polygon is equal to  $180^{\circ} (N-2)$ 

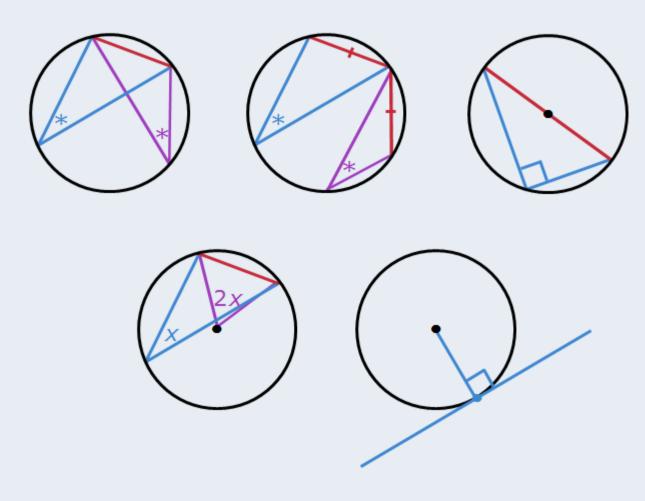




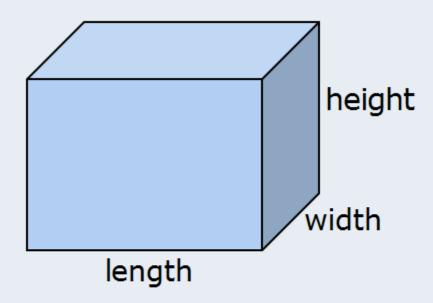




# **Circle Properties**



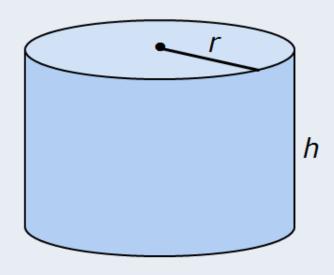




 $volume = length \times width \times height$ 

surface area = sum of areas of all 6 sides





volume = 
$$\pi r^2 h$$

surface area = 
$$\pi r^2 + \pi r^2 + 2\pi rh$$
  
=  $2\pi r^2 + 2\pi rh$   
=  $2\pi r (r + h)$ 



### Conversions

- If conversion is required, relationship will be given
  - e.g., (1 kilometer = 1000 meters)
  - e.g., (1 mile = 5280 feet)
- Note: Relationships not given for units of time
  - e.g., (1 hour = 60 minutes)
  - e.g., (1 day = 24 hours)



### **Geometry Strategies**

- Redraw figures
- Add all given information
- · Add any information that can be deduced
- Add/extend lines
- Assign variables and use algebra
- Problem solving questions drawn to scale:
  - estimate to confirm calculations and guide guesses
- Two or more triangles and length required
  - look for similar triangles
- Right triangle:
  - use Pythagorean Theorem to relate sides
  - watch for Pythagorean Triples and special triangles
- Circle:
  - beware of circle properties (inscribed/central angles, tangent lines)
  - look for isosceles triangles
- Break areas/volumes into manageable pieces



If x and y are integers then:

```
"x is divisible by y" = "when x is divided by y the remainder is 0"

= "y is a divisor of x"

= "y is a factor of x"

= "x equals ky for some integer k"

= "x is a multiple of y"
```

GMAT questions typically focus on **positive** divisors/factors



# **Divisibility Rules**

Di	visible by	Characteristic
	2	Units digit is 0, 2, 4, 6, or 8
	3	Sum of digits is divisible by 3
	4	2-digit # at the end is divisible by 4
	5	Units digit is 0 or 5
	6	The number is divisible by 2 AND by 3
	9	Sum of digits is divisible by 9
	10	Units digit is 0



### **Prime Number**

A positive integer with exactly 2 positive divisors.

Primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, . . .

### Note:

- 1 is not prime (only 1 positive divisor)
- 2 is the **only** even prime number



```
"x is divisible by y'' = "when x is divided by y the remainder is 0"
                            = "y is a divisor of x"
                            = "y is a factor of x"
                            = "x equals ky for some integer k"
                            = "x is a multiple of y"
                            = "y is 'hiding' in the prime factorization of x"
    6 is a divisor of W \Rightarrow W = 6 \times ? \times ? \times ? \times \cdots
                                       =2\times3\times2\times2\times2\times\cdots
    R is divisible by 88 \Rightarrow R = 88 \times ? \times ? \times ? \times ....
                                       = 2 \times 2 \times 2 \times 11 \times 2 \times 2 \times 2 \times \cdots
```



- The **prime factorization** of each number is unique
- Prime factorization can help determine whether numbers are divisors
- If some number, k, is "hiding" in the prime factorization of a number, then k is a divisor of that number



# **Counting Divisors of Large Numbers**

If  $N = p^a \times q^b \times r^c \times \cdots$ , where p, q, r (etc) are prime numbers, then the total number of positive divisors of N is  $(a+1)(b+1)(c+1)\cdots$ 



## **Squares of Integers**

- The prime factorization of a perfect square will have an <u>even</u> number of each prime.
- A perfect square will have an <u>odd</u> number of positive divisors



### **Divisor Rules**

Given: j, k, M and N are integers:

- If k is a divisor of N, then k is a divisor of NM
- If jk is a divisor of N, then j is a divisor of N, and k is a divisor of N
- If k is not a divisor of N, then jk is not a divisor of N
- If k is a divisor of both N and M, then k is a divisor of N+M
   (and N-M and M-N)
- If k is a divisor N, but k is not a divisor of M, then k is not a
  divisor of N+M (or N-M or M-N)



### **Greatest Common Divisor**

Find the greatest common divisor of 56 and 70:

$$56 = 2 \times 2 \times 2 \times 7$$

$$70 = 2 \times 5 \times 7$$

$$GCD = 2 \times 7$$

$$= 14$$

Find the greatest common divisor of 132, 198 and 330:

$$132 = 2 \times 2 \times 3 \times 11$$

$$198 = 2 \times 3 \times 3 \times 11$$

$$330 = 2 \times 3 \times 5 \times 11$$

$$GCD = 2 \times 3 \times 11$$

$$= 66$$



## **Least Common Multiple**

Find the least common multiple of 12 and 56:

$$12 = 2 \times 2 \times 3$$

$$56 = 2 \times 2 \times 2 \times 7$$

$$\downarrow \qquad \qquad \downarrow$$

$$LCM = 2 \times 2 \times 3 \times 2 \times 7$$

$$= 168$$

Find the least common multiple of 18 and 42:

$$18 = 2 \times 3 \times 3$$

$$42 = 2 \times 3 \times 7$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$LCM = 2 \times 3 \times 3 \times 7$$

$$= 126$$



(GCD of x and y)(LCM of x and y) = 
$$xy$$



 $odd \pm odd = even$   $odd \pm even = odd$   $even \pm even = even$ 

$$odd \times odd = odd$$
  
 $odd \times even = even$   
 $even \times even = even$ 

 $\frac{\text{even}}{\text{even}} \text{ can be a non-integer, even or odd}$  If  $\frac{\text{even}}{\text{odd}} \text{ is an integer, it will be even}$   $\frac{\text{odd}}{\text{even}} \text{ cannot be an integer}$  If  $\frac{\text{odd}}{\text{odd}} \text{ is an integer, it will be odd}$ 



(watch the entire video <a href="here">here</a>)

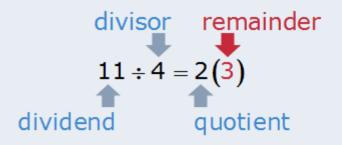
- Create a table to test cases
  - Use "E" and "O" and even/odd rules
  - Plug in values and evaluate
- Draw conclusions based on outcomes



Every  $n^{th}$  integer is divisible by n

n consecutive integers  $\Rightarrow$  1 number must be divisible by n





0 ≤ remainder < divisor

remainder

If  $N \div D = Q(R)$ , then the possible values of N are: R, R + D, R + 2D, R + 3D,...

If 
$$N \div D = Q(R) \implies Q \times D + R = N$$
remainder



$$average = mean = \frac{sum of n numbers}{n}$$

sum of n numbers = (mean)(n)

<u>median</u>: the middlemost value when the numbers are arranged in ascending order

n is odd: median = middle number

n is even: median = average of the 2 middle numbers

mode: the number that occurs most frequently



If the numbers in a set are evenly spaced, then the mean and median of that set are equal

If the mean and median of a set are equal, then the numbers in that set **may or may not** be evenly spaced



Weighted average = 
$$(proportion)\begin{pmatrix} group \\ A \\ average \end{pmatrix} + (proportion)\begin{pmatrix} group \\ B \\ average \end{pmatrix} + \cdots$$

Group A average = a

Group B average = b

case 1) population A = population B

 $\Rightarrow$  average of combined group =  $\frac{a+b}{2}$ 

case 2) population A is greater than population B

average of combined group is closer to a

case 3) population B is greater than population A

→ average of combined group is closer to b



### range = greatest value - least value

Standard Deviation of  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ , . . . ,  $x_n$ 

m = mean

n = number of values

SD = 
$$\sqrt{\frac{(x_1 - m)^2 + (x_2 - m)^2 + (x_3 - m)^2 + \cdots + (x_n - m)^2}{n}}$$

#### Informal definition

**Standard deviation** is the average distance the data values are away from the mean.



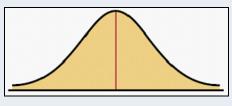
variance = (standard deviation)<sup>2</sup>

## units of standard deviation

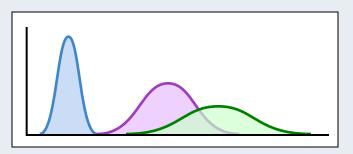
If the standard deviation of a set of numbers is k, then k = 1 unit of standard deviation



#### **Features of Normal Distributions**



- Data are reasonably symmetrical about the mean
- Mean, median and mode are all nearly equal
- About 68% of the data are within 1 standard deviation of the mean
- About 95% of the data are within
   2 standard deviations of the mean
- About 99% of the data are within 3 standard deviations of the mean
- The greater the standard deviation, the wider the curve

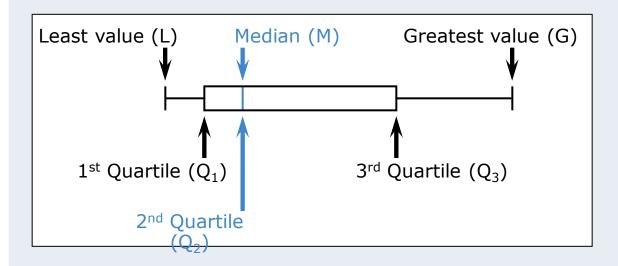




#### **Creating a Boxplot**

- 1) Rearrange values in ascending order
- 2) The median of all values is the  $2^{nd}$  quartile  $(Q_2)$
- 3) The median of the "lesser numbers" is the  $1^{st}$  quartile  $(Q_1)$
- 4) The median of the "greater numbers" is the  $3^{rd}$  quartile ( $Q_3$ )

Exclude Q<sub>2</sub> from the lesser and greater numbers





When tackling counting questions, consider listing and counting.



### **Fundamental Counting Principle**

If a task is comprised of stages, where one stage can be accomplished in A ways, another stage can be accomplished in B ways, another stage can be accomplished in C ways, and so on,

then the total number of ways to accomplish the task is  $A \times B \times C \times \cdots$ 

Can I take the task of "building" possible outcomes and break it into individual stages?



# Arranging n Unique Objects

n unique objects can be arranged in n! ways

### **Factorial Notation**

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$n! = n \times (n-1) \times (n-2) \times \cdot \cdot \cdot \times 3 \times 2 \times 1$$

$$0! = 1$$



- Counting question with restrictions
  - Adhere to the restriction
  - Apply the Restrictions Rule



 Restrictions Rule is useful for questions involving "at least" and "at most"



### MISSISSIPPI Rule

When arranging objects, determine whether the objects are <u>unique</u>

## Arranging Objects When Some are Alike

Given *n* objects where *A* are alike, another *B* are alike, another *C* are alike and so on, the number of ways to arrange the *n* objects is

$$\frac{n!}{(A!)(B!)(C!)\cdots}$$



**Combination**: A selection from a set of unique objects where the order of the selected objects does not matter.

· Choosing committee members is a popular theme

#### **Combination Formula**

r objects can be selected from a set of n unique objects in  ${}_{n}C_{r}$  ways, where:

$${}_{n}^{c}C_{r} = {n \choose r} = \frac{n!}{r!(n-r)!}$$



### When to use Combinations

## General Strategy

- If possible, break the required task into stages
- "Does the outcome of each stage differ from the outcomes of other stages?"

No Combination

Yes - Fundamental Counting Principle or other strategy



## **Calculating Combinations Shortcut**

r objects can be selected from a set of n unique objects in  ${}_{n}C_{r}$  ways

$$_{n}C_{r} = \frac{\text{first } r \text{ values of } n!}{r!}$$



## **Counting Strategies**

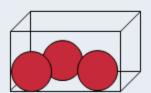
- 1. List outcomes
- 2. If restrictions exist, consider applying the Restrictions Rule
- 3. If possible, break the required task into stages
- 4. Ask, "Do the outcomes of each stage differ from the outcomes of other stages?"
  - No combination
  - Yes continue below
- Determine the number of ways to accomplish each stage, beginning with the most restrictive stage(s)
- 6. Apply the Fundamental Counting Principle
- 7. Arranging objects that are not unique may require the MISSISSIPPI Rule



- The probability of an event = the likelihood that the event will occur
- 0 ≤ probability of an event ≤ 1

```
P(Event A) = 0 
Event A will not occur
```

P(Event A) = 1 - Event A will definitely occur



P(selected ball is green) = 0

P(selected ball is red) = 1



### **Probability of an Event**

In an experiment where each outcome is equally likely, the probability that event A will occur is:

 $P(A) = \frac{\text{number of outcomes where A occurs}}{\text{total number of possible outcomes}}$ 

Calculating the denominator first will often help you gain insight into a question



## Complement

P(event happens) = 1 - P(event DOES NOT happen)

### Possible Uses

- When calculating P(event DOES NOT happen) is easier
- Questions with "at least" and "at most"

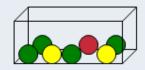


## **Mutually Exclusive Events**

Two events are mutually exclusive if both events cannot occur together.

### **Example**

A ball is randomly selected from the box



Event A: The ball is red

Event B: The ball is yellow

Can both events occur together?

No The events are mutually exclusive

$$\rightarrow$$
  $P(A \text{ and } B) = 0$ 



# "or" Probabilities

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If events A and B are mutually exclusive, then P(A and B) = 0

Events A and B are mutually exclusive

$$ightharpoonup$$
 P(A or B) = P(A) + P(B)



$$P(A AND B) = P(A) \times P(B|A)$$

P(B|A) = probability of event B given that event A has occurred



Events A and B are **dependent** if the occurrence of one event <u>affects</u> the probability of the other. If Events A and B are dependent, then:

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Events A and B are **independent** if the occurrence of one event <u>does not affect</u> the probability of the other. If Events A and B are independent, then:

$$P(A \text{ and } B) = P(A) \times P(B)$$



## **Rewriting Questions**

"or" Probabilities  

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Complement  
$$P(A) = 1 - P(NOT A)$$

## Dependent Events $P(A \text{ and } B) = P(A) \times P(B|A)$

Independent Events  
$$P(A \text{ and } B) = P(A) \times P(B)$$

#### Questions

- What must occur to get this outcome?
- Will it be faster to use the complement?
- Are the events mutually exclusive?
- Are the events independent?



## **General Probability Strategies**

- Consider using the complement
- Determine the general approach
  - Basic probability formula
  - Probability rules
- Basic probability formula
  - Equally likely outcomes?
  - List or use counting techniques
- Probability rules
  - Rewrite question by asking, "What must occur?"
  - "or" probability is test for mutually exclusivity
  - "and" probability in test for independence



## **Guessing Strategies**

- Use your instincts
- For questions that may involve the complement, eliminate any answer choice that does not combine with another answer choice to add to 1



### Strategy

- 1. Understand the big picture
  - Read any accompanying text
  - Pay close attention to units of measurement
  - For graphics with axes:
    - read axis labels
    - determine whether each axis begins at zero
    - determine whether values increase at constant intervals
  - Try to identify possible trends and relationships
    - spike, level out, cyclical?
    - one factor influences another?
- 2. Carefully read the question
  - Beware of discrepancies between units in the text and the units in the data
- Check the answer choices (if there are answer choices) before performing any calculations
  - Indicate the correct form of the answer
  - Indicate the required degree of accuracy



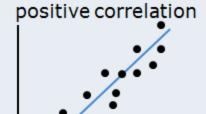
(watch the entire video <a href="here">here</a>)

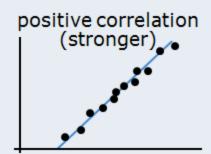
## <u>Tips</u>

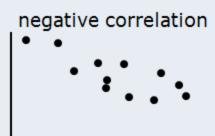
- Estimate whenever possible
- All visual graphics are <u>drawn to scale</u>
  - Visually estimate/compare data
- Do not confuse numbers with rates or percents

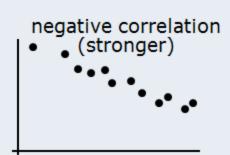


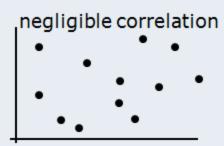
### **Scatter Plots**













#### **Scatter Plots**

- Analyze relationships between two shared variables in a population
- Trend line (regression line): line that best fits the data points
- The closer the points to the trend line, the stronger the correlation
- Positive correlation: one value ↑ as the other ↑
- ullet Negative correlation: one value ullet as the other ullet
- Negligible correlation: little or no relationship between variables

